Once you have trade consulates your diplomatic controls expand. You are able to establish embassies, and set trade policies with the trade consulate nations.

The trade policies tab is used to grant trade subsidies to the nations where you have trade consulates. This will encourage those nations to trade with you first. As each trade deal is made relations between your country and the Minor Nation will improve further.

The diplomatic overtures tab is used to establish embassies.

Embassies immensely increase your diplomatic options and pave the way for eventual peaceful investment in and colonizing of the Minor Nation.

Once you have an Embassy you may use the grants tab to offer money to the other nation's leaders. These bribes lead to rapid improvement in relations. It is best not to question how this money is spent.